FIFTH EDITION

Precalculus with Limits A Graphing Approach

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Cover photograph
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Library of Congress Control Number: 2006930929
Student Edition:
ISBN-13: 978-0-618-85152-2
ISBN-10: 0-618-85152-6

Brooks/Cole
10 Davis Drive
Belmont, CA 94002-3098
USA

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Printed in the United States of America
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A Word from the Authors

Welcome to *Precalculus with Limits: A Graphing Approach*, Fifth Edition. We are pleased to present this new edition of our textbook in which we focus on making the mathematics accessible, supporting student success, and offering teachers flexible teaching options.

Accessible to Students

We have taken care to write this text with the student in mind. Paying careful attention to the presentation, we use precise mathematical language and a clear writing style to develop an effective learning tool. We believe that every student can learn mathematics, and we are committed to providing a text that makes the mathematics of the precalculus course accessible to all students.

Throughout the text, solutions to many examples are presented from multiple perspectives—algebraically, graphically, and numerically. The side-by-side format of this pedagogical feature helps students to see that a problem can be solved in more than one way and to see that different methods yield the same result. The side-by-side format also addresses many different learning styles.

We have found that many precalculus students grasp mathematical concepts more easily when they work with them in the context of real-life situations. Students have numerous opportunities to do this throughout this text. The *Make a Decision* feature further connects real-life data and applications and motivates students. It also offers students the opportunity to generate and analyze mathematical models from large data sets. To reinforce the concept of functions, we have compiled all the elementary functions as a *Library of Parent Functions*, presented in a summary on the endpapers of the text for convenient reference. Each function is introduced at the first point of use in the text with a definition and description of basic characteristics.

We have carefully written and designed each page to make the book more readable and accessible to students. For example, to avoid unnecessary page turning and disruptions to students' thought processes, each example and corresponding solution begins and ends on the same page.

Supports Student Success

During more than 30 years of teaching and writing, we have learned many things about the teaching and learning of mathematics. We have found that students are most successful when they know what they are expected to learn and why it is important to learn the concepts. With that in mind, we have incorporated a thematic study thread throughout this textbook.

Each chapter begins with a list of applications that are covered in the chapter and serve as a motivational tool by connecting section content to real-life situations. Using the same pedagogical theme, each section begins with a set of section learning objectives—*What You Should Learn*. These are followed by an engaging real-life application—*Why You Should Learn It*—that motivates students and illustrates an area where the mathematical concepts will be applied in an example or exercise in the section. The *Chapter Summary—What Did You Learn?*—at the end of each chapter includes *Key Terms* with page references and *Key Concepts*, organized by section, that were covered throughout the chapter. The *Chapter Summary* serves as a useful study aid for students.

Throughout the text, other features further improve accessibility. *Study Tips* are provided throughout the text at point-of-use to reinforce concepts and to help students learn how to study mathematics. *Explorations* reinforce mathematical concepts. Each example with worked-out solution is followed by a *Checkpoint*, which directs the student to work a similar exercise from the exercise set. The *Section Exercises* begin with a *Vocabulary Check*, which gives the students an opportunity to test their understanding of the important terms in the section. A *Prerequisites Skills* is offered in margin notes throughout the textbook exposition. Reviewing the prerequisite skills will enable students to master new concepts more quickly. *Synthesis Exercises* check students' conceptual understanding of the topics in each section. *Skills Review Exercises* provide additional practice with the concepts in the chapter or previous chapters. *Review Exercises*, *Chapter Tests*, and periodic *Cumulative Tests* offer students frequent opportunities for self-assessment and to develop strong study and test-taking skills. The *Progressive Summaries* and the *Study Capsules* serve as a quick reference when working on homework or as a cumulative study aid.

The use of technology also supports students with different learning styles, and graphing calculators are fully integrated into the text presentation. The *Technology Support Appendix* makes it easier for students to use technology. *Technology Support* notes are provided throughout the text at point-of-use. These notes guide students to the *Technology Support Appendix*, where they can learn how to use specific graphing calculator features to enhance their understanding of the concepts presented in the text. These notes also direct students to the *Graphing Technology Guide*, in the *Online Study Center*, for keystroke support that is available for numerous calculator models. *Technology Tips* are provided in the text at point-of-use to call attention to the strengths and weaknesses of graphing technology, as well as to offer alternative methods for solving or checking a problem using technology. Because students are often misled by the limitations of graphing calculators, we have, where appropriate, used color to enhance the graphing calculator displays in the textbook. This enables students to visualize the mathematical concepts clearly and accurately and avoid common misunderstandings.

Numerous additional text-specific resources are available to help students succeed in the precalculus course. These include "live" online tutoring, instructional DVDs, and a variety of other resources, such as tutorial support and self-assessment, which are available on the Web and in Eduspace®. In addition, the *Online Notetaking Guide* is a notetaking guide that helps students organize their class notes and create an effective study and review tool.

Flexible Options for Teachers

From the time we first began writing textbooks in the early 1970s, we have always considered it a critical part of our role as authors to provide teachers with flexible programs. In addition to addressing a variety of learning styles, the optional features within the text allow teachers to design their courses to meet their instructional needs and the needs of their students. For example, the *Explorations* throughout the text can be used as a quick introduction to concepts or as a way to reinforce student understanding.

Our goal when developing the exercise sets was to address a wide variety of learning styles and teaching preferences. The *Vocabulary Check* questions are provided at the beginning of every exercise set to help students learn proper mathematical terminology. In each exercise set we have included a variety of exercise types, including questions requiring writing and critical thinking, as well as real-data applications. The problems are carefully graded in difficulty from mastery of basic skills to more challenging exercises. Some of the more challenging exercises include the *Synthesis Exercises* that combine skills and are used to check for conceptual understanding, and the *Make a Decision* exercises that further connect real-life data and applications and motivate students. *Skills Review Exercises*, placed at the end of each exercise set, reinforce previously learned skills. The *Proofs in Mathematics*, at the end of each chapter, are proofs of important mathematical properties and theorems and illustrate various proof techniques. This feature gives the teachers the opportunity to incorporate more rigor into their course. In addition, Cengage Learning's Eduspace® website offers teachers the option to assign homework and tests online—and also includes the ability to grade these assignments automatically.

Other print and media resources are available to support teachers. The *Teacher's Edition* provides the complete student text plus point-of-use annotations for teachers in an easy-to-use, wrap-around format. The *Complete Solutions Guide* and its online version offer worked-out solutions to every exercise in the text. For flexibility, the *Test Item File* is available both in print and via Diploma Testing provides teachers all the tools they need to create, author/edit, customize, and deliver multiple types of tests. Teachers can use existing test bank questions, edit the content, and write new static or algorithmic questions all within *Diploma's* powerful electronic platform. *The Electronic Classroom* offers customizable PowerPoint presentations for the classroom. Also included are a variety of transparency masters–including warm-up, daily homework, and answers to section exercises. Finally, the *Online Teaching Center* offers an array of resources provided conveniently via the Web, and the *Online Instructor Success Organizer* is an invaluable aid throughout the year.

Teachers who stress applications and problem-solving, integrating technology into their course will find this program right for them.

Students find extra support in a number of special supplements that reinforce concepts and help them organize their study. First, the *Study and Solutions Guide* provides step-by-step solutions for all odd-numbered text exercises as well as chapter and cumulative tests. The manual also provides practice tests accompanied by a solution key. An invaluable study aid, the *Notetaking Guide* helps students prepare for chapter and/or cumulative tests. It features a lesson-by-lesson framework that allows students to take notes on and review key concepts throughout the text. The *Online Study Center* provides numerous interactive lessons, simulations, animations, and applications, as well as a glossary with flash cards and a graphing calculator program. An abundance of resources are contained in the *Online Study Center*, including the *Online Notetaking Guide*. Finally, Dana Mosely hosts the *Instructional DVDs*. They cover every section in the text, providing clear explanations of key concepts, examples, exercises, and applications in a lecture-based format. New to the DVDs is captioning for the hearing-impaired.

We hope you enjoy the Fifth Edition.

Ron Larson

Robert Hostetler

Bruce H. Edwards

Acknowledgments

We would like to thank the many people who have helped us prepare the text and supplements package, including all those reviewers who have contributed to this and previous editions of the text. Their encouragement, criticisms, and suggestions have been invaluable to us.

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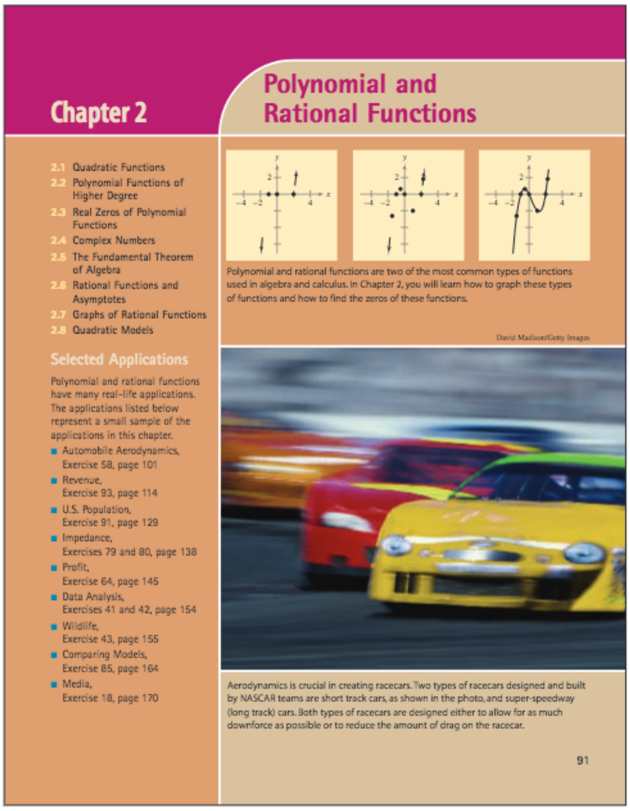
We would like to thank the staff of Larson Texts, Inc. who assisted in preparing the manuscript, rendering the art package, and typesetting and proofreading the pages and supplements.

On a personal level, we are grateful to our wives, Deanna Gilbert Larson, Eloise Hostetler, and Consuelo Edwards for their love, patience, and support. Also, a special thanks goes to R. Scott O'Neil.

If you have suggestions for improving this text, please feel free to write us. Over the past two decades we have received many useful comments from both teachers and students, and we value these very much.

Ron Larson
Robert Hostetler
Bruce H. Edwards

Features Highlights



Chapter 2

Polynomial and Rational Functions

2.1 Quadratic Functions
2.2 Polynomial Functions of Higher Degree
2.3 Real Zeros of Polynomial Functions
2.4 Complex Numbers
2.5 The Fundamental Theorem of Algebra
2.6 Rational Functions and Asymptotes
2.7 Graphs of Rational Functions
2.8 Quadratic Models

Selected Applications

Polynomial and rational functions have many real-life applications. The applications listed below represent a small sample of the applications in this chapter.

- Automobile Aerodynamics, Exercise 58, page 101
- Revenue, Exercise 93, page 114
- U.S. Population, Exercise 91, page 129
- Impedance, Exercises 79 and 80, page 138
- Profit, Exercise 64, page 145
- Data Analysis, Exercises 41 and 42, page 154
- Wildlife, Exercise 43, page 155
- Comparing Models, Exercise 85, page 164
- Media, Exercise 18, page 170

Polynomial and rational functions are two of the most common types of functions used in algebra and calculus. In Chapter 2, you will learn how to graph these types of functions and how to find the zeros of these functions.

David Madison/Getty Images

Aerodynamics is crucial in creating racecars. Two types of racecars designed and built by NASCAR teams are short track cars, as shown in the photo, and super-speedway (long track) cars. Both types of racecars are designed either to allow for as much downforce as possible or to reduce the amount of drag on the racecar.

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Chapter Opener

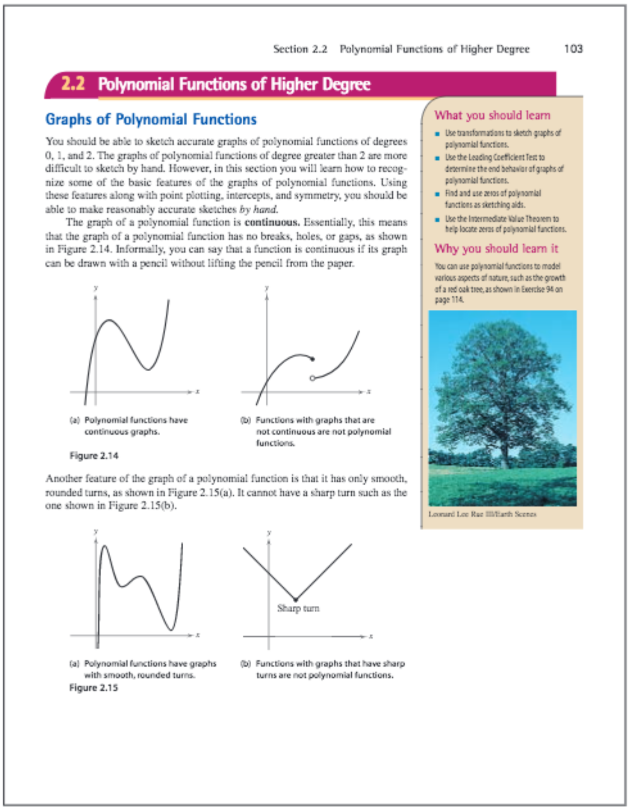
Each chapter begins with a comprehensive overview of the chapter concepts. The photograph and caption illustrate a real-life application of a key concept. Section references help students prepare for the chapter.

Applications List

An abridged list of applications, covered in the chapter, serve as a motivational tool by connecting section content to real-life situations.

"What You Should Learn" and "Why You Should Learn It"

Sections begin with *What You Should Learn*, an outline of the main concepts covered in the section, and *Why You Should Learn It*, a real-life application or mathematical reference that illustrates the relevance of the section content.



Section 2.2 Polynomial Functions of Higher Degree 103

2.2 Polynomial Functions of Higher Degree

Graphs of Polynomial Functions

You should be able to sketch accurate graphs of polynomial functions of degrees 0, 1, and 2. The graphs of polynomial functions of degree greater than 2 are more difficult to sketch by hand. However, in this section you will learn how to recognize some of the basic features of the graphs of polynomial functions. Using these features along with point plotting, intercepts, and symmetry, you should be able to make reasonably accurate sketches *by hand.*

The graph of a polynomial function is **continuous.** Essentially, this means that the graph of a polynomial function has no breaks, holes, or gaps, as shown in Figure 2.14. Informally, you can say that a function is continuous if its graph can be drawn with a pencil without lifting the pencil from the paper.

(a) Polynomial functions have continuous graphs. (b) Functions with graphs that are not continuous are not polynomial functions.

Figure 2.14

Another feature of the graph of a polynomial function is that it has only smooth, rounded turns, as shown in Figure 2.15(a). It cannot have a sharp turn such as the one shown in Figure 2.15(b).

Sharp turn

(a) Polynomial functions have graphs with smooth, rounded turns. (b) Functions with graphs that have sharp turns are not polynomial functions.

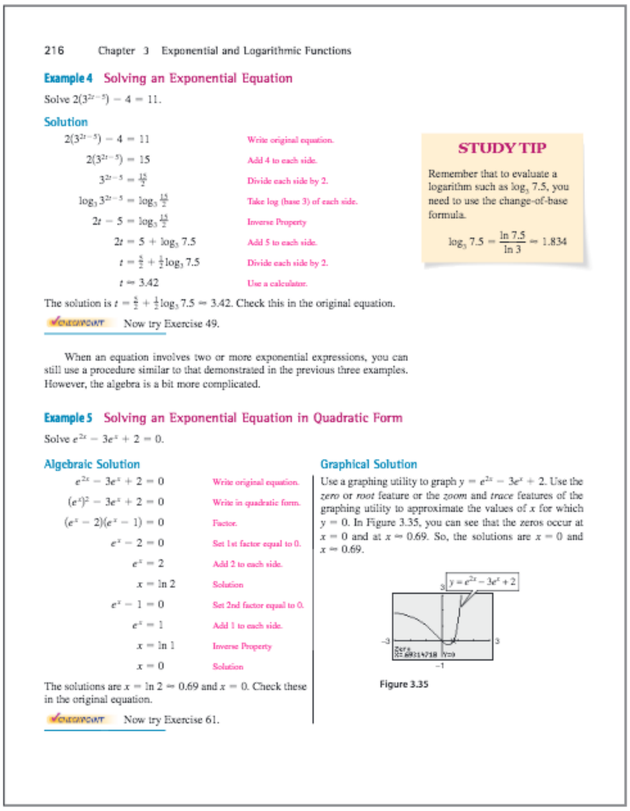
Figure 2.15

What you should learn
- Use transformations to sketch graphs of polynomial functions.
- Use the Leading Coefficient Test to determine the end behavior of graphs of polynomial functions.
- Find and use zeros of polynomial functions as sketching aids.
- Use the Intermediate Value Theorem to help locate zeros of polynomial functions.

Why you should learn it

You can use polynomial functions to model various aspects of nature, such as the growth of a red oak tree, as shown in Exercise 94 on page 114.

Leonard Lee Rue III/Earth Scenes



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Example 4 Solving an Exponential Equation

Solve $2(3^{2t-5}) - 4 = 11$.

Solution

$2(3^{2t-5}) - 4 = 11$ Write original equation.

$2(3^{2t-5}) = 15$ Add 4 to each side.

$3^{2t-5} = \frac{15}{2}$ Divide each side by 2.

$\log_3 3^{2t-5} = \log_3 \frac{15}{2}$ Take log (base 3) of each side.

$2t - 5 = \log_3 \frac{15}{2}$ Inverse Property

$2t = 5 + \log_3 7.5$ Add 5 to each side.

$t = \frac{5}{2} + \frac{1}{2}\log_3 7.5$ Divide each side by 2.

$t \approx 3.42$ Use a calculator.

The solution is $t = \frac{5}{2} + \frac{1}{2}\log_3 7.5 \approx 3.42$. Check this in the original equation.

CHECKPOINT Now try Exercise 49.

STUDY TIP

Remember that to evaluate a logarithm such as $\log_3 7.5$, you need to use the change-of-base formula.

$$\log_3 7.5 = \frac{\ln 7.5}{\ln 3} \approx 1.834$$

When an equation involves two or more exponential expressions, you can still use a procedure similar to that demonstrated in the previous three examples. However, the algebra is a bit more complicated.

Example 5 Solving an Exponential Equation in Quadratic Form

Solve $e^{2x} - 3e^x + 2 = 0$.

Algebraic Solution

$e^{2x} - 3e^x + 2 = 0$ Write original equation.

$(e^x)^2 - 3e^x + 2 = 0$ Write in quadratic form.

$(e^x - 2)(e^x - 1) = 0$ Factor.

$e^x - 2 = 0$ Set 1st factor equal to 0.

$e^x = 2$ Add 2 to each side.

$x = \ln 2$ Solution

$e^x - 1 = 0$ Set 2nd factor equal to 0.

$e^x = 1$ Add 1 to each side.

$x = \ln 1$ Inverse Property

$x = 0$ Solution

The solutions are $x = \ln 2 \approx 0.69$ and $x = 0$. Check these in the original equation.

CHECKPOINT Now try Exercise 61.

Graphical Solution

Use a graphing utility to graph $y = e^{2x} - 3e^x + 2$. Use the *zero* or *root* feature or the *zoom* and *trace* features of the graphing utility to approximate the values of x for which $y = 0$. In Figure 3.35, you can see that the zeros occur at $x = 0$ and at $x \approx 0.69$. So, the solutions are $x = 0$ and $x \approx 0.69$.

Figure 3.35

Examples

Many examples present side-by-side solutions with multiple approaches—algebraic, graphical, and numerical. This format addresses a variety of learning styles and shows students that different solution methods yield the same result.

Checkpoint

The *Checkpoint* directs students to work a similar problem in the exercise set for extra practice.

Study Tips

Study Tips reinforce concepts and help students learn how to study mathematics.

Library of Parent Functions

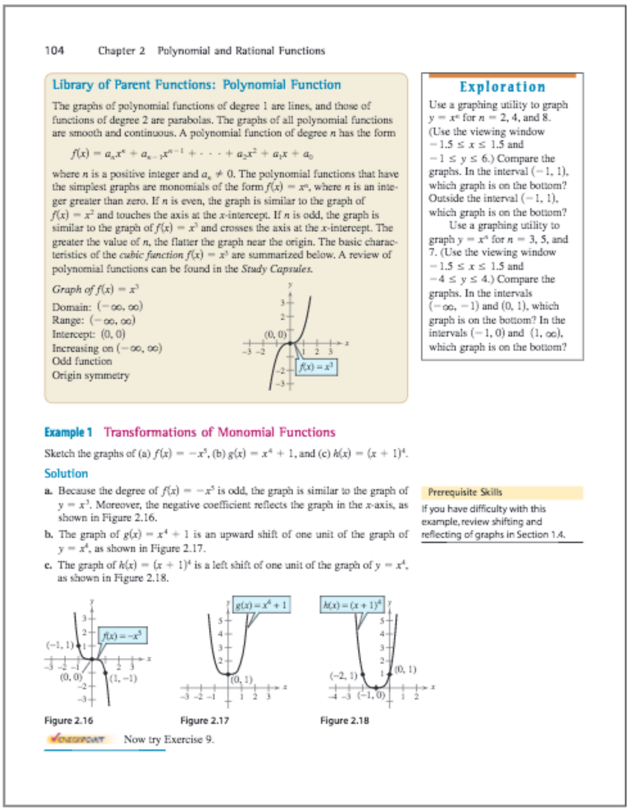
The *Library of Parent Functions* feature defines each elementary function and its characteristics at first point of use. The *Study Capsules* are also referenced for further review of each elementary function.

Explorations

The *Explorations* engage students in active discovery of mathematical concepts, strengthen critical thinking skills, and help them to develop an intuitive understanding of theoretical concepts.

New! Prerequisite Skills

A review of algebra skills needed to complete the examples is offered to the students at point of use throughout the text.



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Library of Parent Functions: Polynomial Function

The graphs of polynomial functions of degree 1 are lines, and those of functions of degree 2 are parabolas. The graphs of all polynomial functions are smooth and continuous. A polynomial function of degree n has the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$$

where n is a positive integer and $a_n \neq 0$. The polynomial functions that have the simplest graphs are monomials of the form $f(x) = x^n$, where n is an integer greater than zero. If n is even, the graph is similar to the graph of $f(x) = x^2$ and touches the axis at the x-intercept. If n is odd, the graph is similar to the graph of $f(x) = x^3$ and crosses the axis at the x-intercept. The greater the value of n, the flatter the graph near the origin. The basic characteristics of the *cubic function* $f(x) = x^3$ are summarized below. A review of polynomial functions can be found in the *Study Capsules*.

Graph of $f(x) = x^3$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Intercept: $(0, 0)$
Increasing on $(-\infty, \infty)$
Odd function
Origin symmetry

Exploration

Use a graphing utility to graph $y = x^n$ for $n = 2, 4$, and 8. (Use the viewing window $-1.5 \le x \le 1.5$ and $-1 \le y \le 6$.) Compare the graphs. In the interval $(-1, 1)$, which graph is on the bottom? Outside the interval $(-1, 1)$, which graph is on the bottom?

Use a graphing utility to graph $y = x^n$ for $n = 3, 5$, and 7. (Use the viewing window $-1.5 \le x \le 1.5$ and $-4 \le y \le 4$.) Compare the graphs. In the intervals $(-\infty, -1)$ and $(0, 1)$, which graph is on the bottom? In the intervals $(-1, 0)$ and $(1, \infty)$, which graph is on the bottom?

Example 1 Transformations of Monomial Functions

Sketch the graphs of (a) $f(x) = -x^5$, (b) $g(x) = x^4 + 1$, and (c) $h(x) = (x + 1)^4$.

Solution

a. Because the degree of $f(x) = -x^5$ is odd, the graph is similar to the graph of $y = x^3$. Moreover, the negative coefficient reflects the graph in the x-axis, as shown in Figure 2.16.

b. The graph of $g(x) = x^4 + 1$ is an upward shift of one unit of the graph of $y = x^4$, as shown in Figure 2.17.

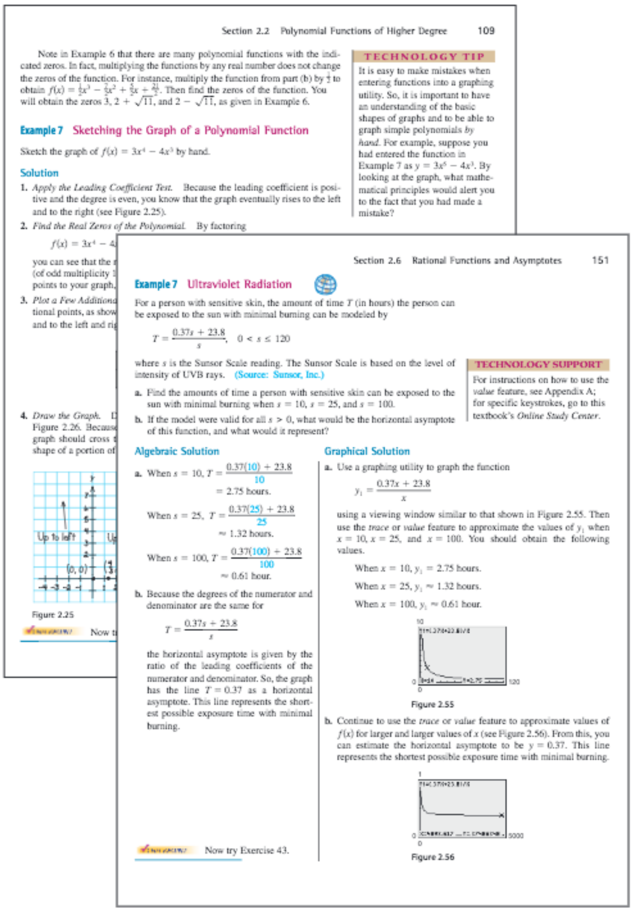
c. The graph of $h(x) = (x + 1)^4$ is a left shift of one unit of the graph of $y = x^4$, as shown in Figure 2.18.

Prerequisite Skills

If you have difficulty with this example, review shifting and reflecting of graphs in Section 1.4.

Figure 2.16 Figure 2.17 Figure 2.18

CHECKPOINT Now try Exercise 9.

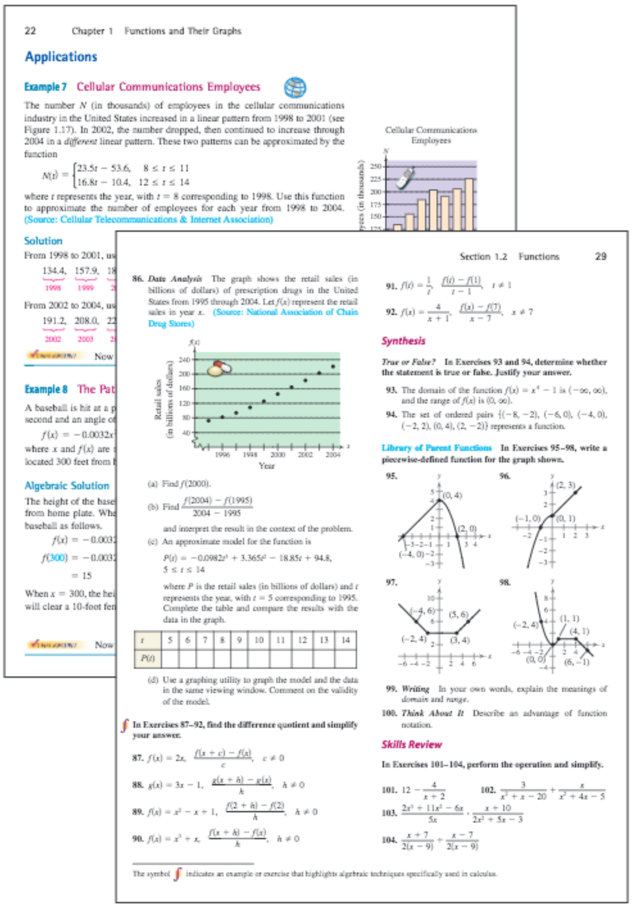


Technology Tip

Technology Tips point out the pros and cons of technology use in certain mathematical situations. *Technology Tips* also provide alternative methods of solving or checking a problem by the use of a graphing calculator.

Technology Support

The *Technology Support* feature guides students to the *Technology Support Appendix* if they need to reference a specific calculator feature. These notes also direct students to the *Graphing Technology Guide*, in the *Online Study Center*, for keystroke support that is available for numerous calculator models.

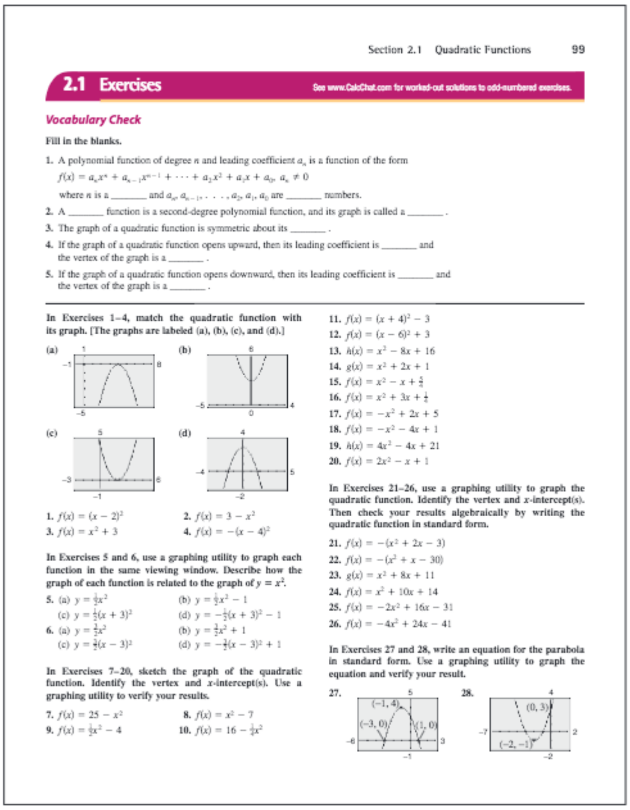


Real-Life Applications

A wide variety of real-life applications, many using current real data, are integrated throughout the examples and exercises. The ▢ indicates an example that involves a real-life application.

Algebra of Calculus

Throughout the text, special emphasis is given to the algebraic techniques used in calculus. *Algebra of Calculus* examples and exercises are integrated throughout the text and are identified by the symbol ▢.



Section 2.1 Quadratic Functions 99

2.1 Exercises See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Vocabulary Check

Fill in the blanks.

1. A polynomial function of degree n and leading coefficient a_n is a function of the form
$f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0,\ a_n \neq 0$
where n is a _______ and $a_n, a_{n-1}, \ldots, a_2, a_1, a_0$ are _______ numbers.
2. A _______ function is a second-degree polynomial function, and its graph is called a _______ .
3. The graph of a quadratic function is symmetric about its _______ .
4. If the graph of a quadratic function opens upward, then its leading coefficient is _______ and the vertex of the graph is a _______ .
5. If the graph of a quadratic function opens downward, then its leading coefficient is _______ and the vertex of the graph is a _______ .

In Exercises 1–4, match the quadratic function with its graph. [The graphs are labeled (a), (b), (c), and (d).]

(a) (b) (c) (d)

1. $f(x) = (x-2)^2$
2. $f(x) = 3 - x^2$
3. $f(x) = x^2 + 3$
4. $f(x) = -(x-4)^2$

In Exercises 5 and 6, use a graphing utility to graph each function in the same viewing window. Describe how the graph of each function is related to the graph of $y = x^2$.

5. (a) $y = \frac{1}{2}x^2$ (b) $y = \frac{1}{2}x^2 - 1$
(c) $y = \frac{1}{2}(x+3)^2$ (d) $y = -\frac{1}{2}(x+3)^2 - 1$
6. (a) $y = \frac{3}{2}x^2$ (b) $y = \frac{3}{2}x^2 + 1$
(c) $y = \frac{3}{2}(x-3)^2$ (d) $y = -\frac{3}{2}(x-3)^2 + 1$

In Exercises 7–20, sketch the graph of the quadratic function. Identify the vertex and x-intercept(s). Use a graphing utility to verify your results.

7. $f(x) = 25 - x^2$
8. $f(x) = x^2 - 7$
9. $f(x) = \frac{1}{2}x^2 - 4$
10. $f(x) = 16 - \frac{1}{4}x^2$
11. $f(x) = (x+4)^2 - 3$
12. $f(x) = (x-6)^2 + 3$
13. $h(x) = x^2 - 8x + 16$
14. $g(x) = x^2 + 2x + 1$
15. $f(x) = x^2 - x + \frac{5}{4}$
16. $f(x) = x^2 + 3x + \frac{1}{4}$
17. $f(x) = -x^2 + 2x + 5$
18. $f(x) = -x^2 - 4x + 1$
19. $h(x) = 4x^2 - 4x + 21$
20. $f(x) = 2x^2 - x + 1$

In Exercises 21–26, use a graphing utility to graph the quadratic function. Identify the vertex and x-intercept(s). Then check your results algebraically by writing the quadratic function in standard form.

21. $f(x) = -(x^2 + 2x - 3)$
22. $f(x) = -(x^2 + x - 30)$
23. $g(x) = x^2 + 8x + 11$
24. $f(x) = x^2 + 10x + 14$
25. $f(x) = -2x^2 + 16x - 31$
26. $f(x) = -4x^2 + 24x - 41$

In Exercises 27 and 28, write an equation for the parabola in standard form. Use a graphing utility to graph the equation and verify your result.

27. (−1, 4) (−3, 0) (1, 0)
28. (0, 3) (−2, −1)

Section Exercises

The section exercise sets consist of a variety of computational, conceptual, and applied problems.

Vocabulary Check

Section exercises begin with a *Vocabulary Check* that serves as a review of the important mathematical terms in each section.

New! Calc Chat

The worked-out solutions to the odd-numbered text exercises are now available at *www.CalcChat.com.*

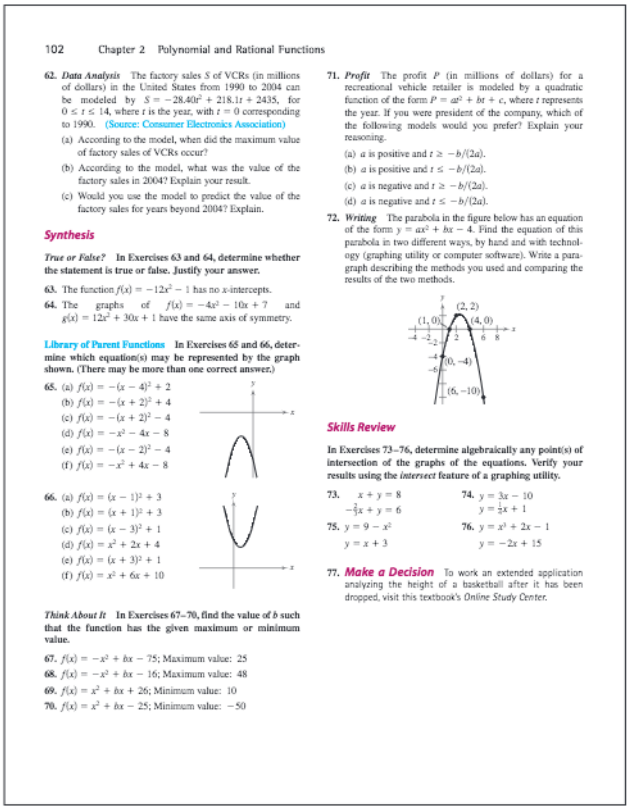
Synthesis and Skills Review Exercises

Each exercise set concludes with three types of exercises.

Synthesis exercises promote further exploration of mathematical concepts, critical thinking skills, and writing about mathematics. The exercises require students to show their understanding of the relationships between many concepts in the section.

Skills Review Exercises reinforce previously learned skills and concepts.

New! *Make a Decision* exercises, found in selected sections, further connect real-life data and applications and motivate students. They also offer students the opportunity to generate and analyze mathematical models from large data sets.



102 Chapter 2 Polynomial and Rational Functions

62. ***Data Analysis*** The factory sales S of VCRs (in millions of dollars) in the United States from 1990 to 2004 can be modeled by $S = -28.40t^2 + 218.1t + 2435$, for $0 \le t \le 14$, where t is the year, with $t = 0$ corresponding to 1990. (Source: Consumer Electronics Association)
(a) According to the model, when did the maximum value of factory sales of VCRs occur?
(b) According to the model, what was the value of the factory sales in 2004? Explain your result.
(c) Would you use the model to predict the value of the factory sales for years beyond 2004? Explain.

Synthesis

True or False? **In Exercises 63 and 64, determine whether the statement is true or false. Justify your answer.**

63. The function $f(x) = -12x^2 - 1$ has no x-intercepts.
64. The graphs of $f(x) = -4x^2 - 10x + 7$ and $g(x) = 12x^2 + 30x + 1$ have the same axis of symmetry.

Library of Parent Functions In Exercises 65 and 66, determine which equation(s) may be represented by the graph shown. (There may be more than one correct answer.)

65. (a) $f(x) = -(x-4)^2 + 2$
(b) $f(x) = -(x+2)^2 + 4$
(c) $f(x) = -(x+2)^2 - 4$
(d) $f(x) = -x^2 - 4x - 8$
(e) $f(x) = -(x-2)^2 - 4$
(f) $f(x) = -x^2 + 4x - 8$

66. (a) $f(x) = (x-1)^2 + 3$
(b) $f(x) = (x+1)^2 + 3$
(c) $f(x) = (x-3)^2 + 1$
(d) $f(x) = x^2 + 2x + 4$
(e) $f(x) = (x+3)^2 + 1$
(f) $f(x) = x^2 + 6x + 10$

Think About It **In Exercises 67–70, find the value of b such that the function has the given maximum or minimum value.**

67. $f(x) = -x^2 + bx - 75$; Maximum value: 25
68. $f(x) = -x^2 + bx - 16$; Maximum value: 48
69. $f(x) = x^2 + bx + 26$; Minimum value: 10
70. $f(x) = x^2 + bx - 25$; Minimum value: −50

71. ***Profit*** The profit P (in millions of dollars) for a recreational vehicle retailer is modeled by a quadratic function of the form $P = at^2 + bt + c$, where t represents the year. If you were president of the company, which of the following models would you prefer? Explain your reasoning.
(a) a is positive and $t \ge -b/(2a)$.
(b) a is positive and $t \le -b/(2a)$.
(c) a is negative and $t \ge -b/(2a)$.
(d) a is negative and $t \le -b/(2a)$.

72. ***Writing*** The parabola in the figure below has an equation of the form $y = ax^2 + bx - 4$. Find the equation of this parabola in two different ways, by hand and with technology (graphing utility or computer software). Write a paragraph describing the methods you used and comparing the results of the two methods.

(2, 2) (1, 0) (4, 0) (0, −4) (6, −10)

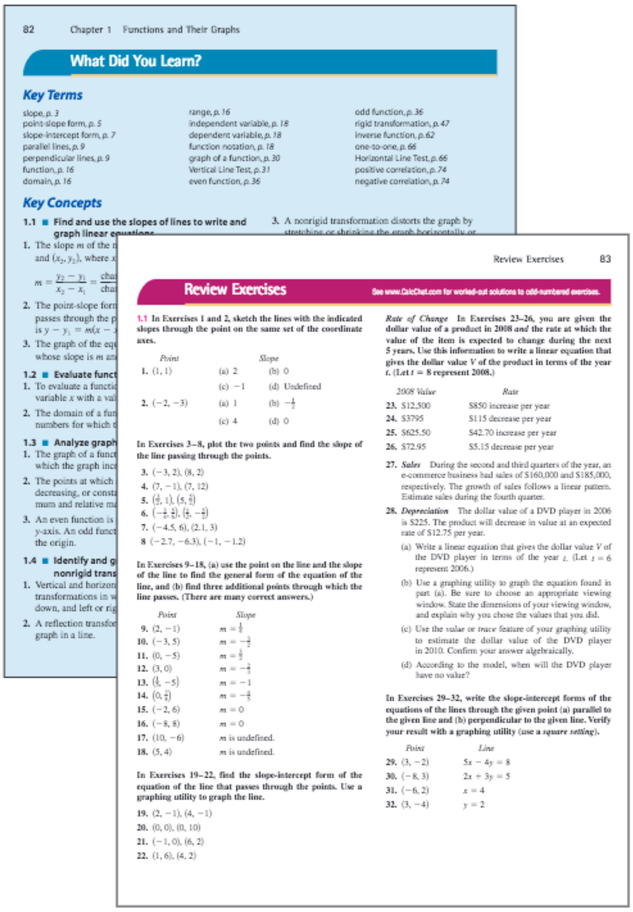
Skills Review

In Exercises 73–76, determine algebraically any point(s) of intersection of the graphs of the equations. Verify your results using the *intersect* feature of a graphing utility.

73. $x + y = 8$
$-\frac{2}{3}x + y = 6$
74. $y = 3x - 10$
$y = \frac{1}{2}x + 1$
75. $y = 9 - x^2$
$y = x + 3$
76. $y = x^3 + 2x - 1$
$y = -2x + 15$

77. ***Make a Decision*** To work an extended application analyzing the height of a basketball after it has been dropped, visit this textbook's *Online Study Center.*

FEATURES



Chapter Summary

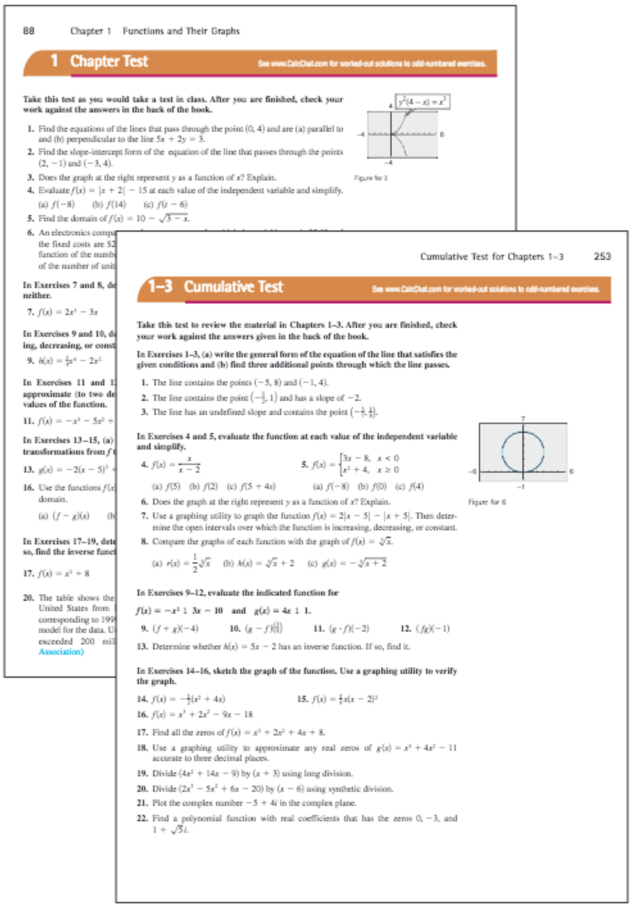
The *Chapter Summary* "*What Did You Learn?*" includes *Key Terms* with page references and *Key Concepts*, organized by section, that were covered throughout the chapter.

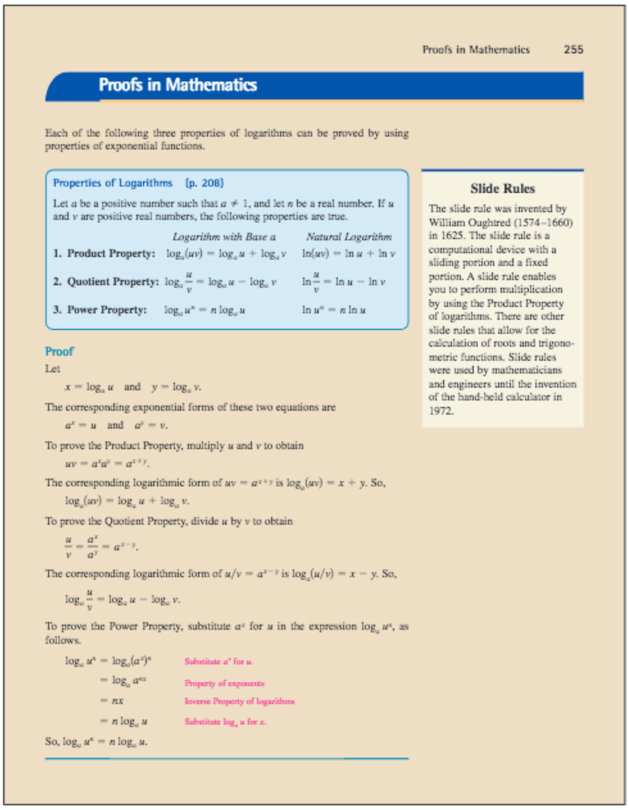
Review Exercises

The chapter *Review Exercises* provide additional practice with the concepts covered in the chapter.

Chapter Tests and Cumulative Tests

Chapter Tests, at the end of each chapter, and periodic *Cumulative Tests* offer students frequent opportunities for self-assessment and to develop strong study and test-taking skills.




Proofs in Mathematics 255

Proofs in Mathematics

Each of the following three properties of logarithms can be proved by using properties of exponential functions.

Properties of Logarithms (p. 208)

Let a be a positive number such that $a \neq 1$, and let n be a real number. If u and v are positive real numbers, the following properties are true.

	Logarithm with Base a	*Natural Logarithm*
1. Product Property:	$\log_a(uv) = \log_a u + \log_a v$	$\ln(uv) = \ln u + \ln v$
2. Quotient Property:	$\log_a \frac{u}{v} = \log_a u - \log_a v$	$\ln \frac{u}{v} = \ln u - \ln v$
3. Power Property:	$\log_a u^n = n \log_a u$	$\ln u^n = n \ln u$

Slide Rules

The slide rule was invented by William Oughtred (1574–1660) in 1625. The slide rule is a computational device with a sliding portion and a fixed portion. A slide rule enables you to perform multiplication by using the Product Property of logarithms. There are other slide rules that allow for the calculation of roots and trigonometric functions. Slide rules were used by mathematicians and engineers until the invention of the hand-held calculator in 1972.

Proof

Let

$x = \log_a u$ and $y = \log_a v$.

The corresponding exponential forms of these two equations are

$a^x = u$ and $a^y = v$.

To prove the Product Property, multiply u and v to obtain

$uv = a^x a^y = a^{x+y}$.

The corresponding logarithmic form of $uv = a^{x+y}$ is $\log_a(uv) = x + y$. So,

$\log_a(uv) = \log_a u + \log_a v$.

To prove the Quotient Property, divide u by v to obtain

$\frac{u}{v} = \frac{a^x}{a^y} = a^{x-y}$.

The corresponding logarithmic form of $u/v = a^{x-y}$ is $\log_a(u/v) = x - y$. So,

$\log_a \frac{u}{v} = \log_a u - \log_a v$.

To prove the Power Property, substitute a^x for u in the expression $\log_a u^n$, as follows.

$\log_a u^n = \log_a (a^x)^n$ Substitute a^x for u.

$= \log_a a^{nx}$ Property of exponents

$= nx$ Inverse Property of logarithms

$= n \log_a u$ Substitute $\log_a u$ for x.

So, $\log_a u^n = n \log_a u$.

Proofs in Mathematics

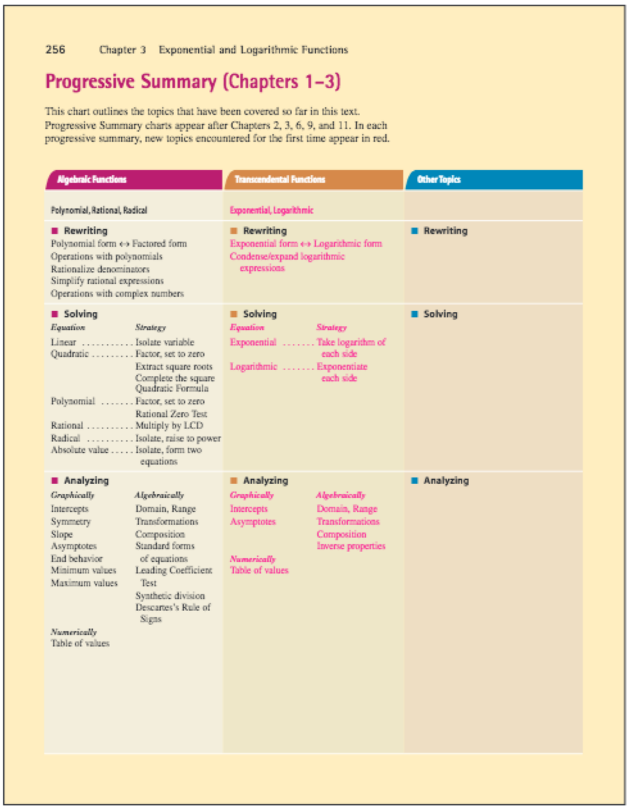
At the end of every chapter, proofs of important mathematical properties and theorems are presented as well as discussions of various proof techniques.

New! Progressive Summaries

The *Progressive Summaries* are a series of charts that are usually placed at the end of every third chapter. Each *Progressive Summary* is completed in a gradual manner as new concepts are covered. Students can use the *Progressive Summaries* as a cumulative study aid and to see the connection between concepts and skills.

New! Study Capsules

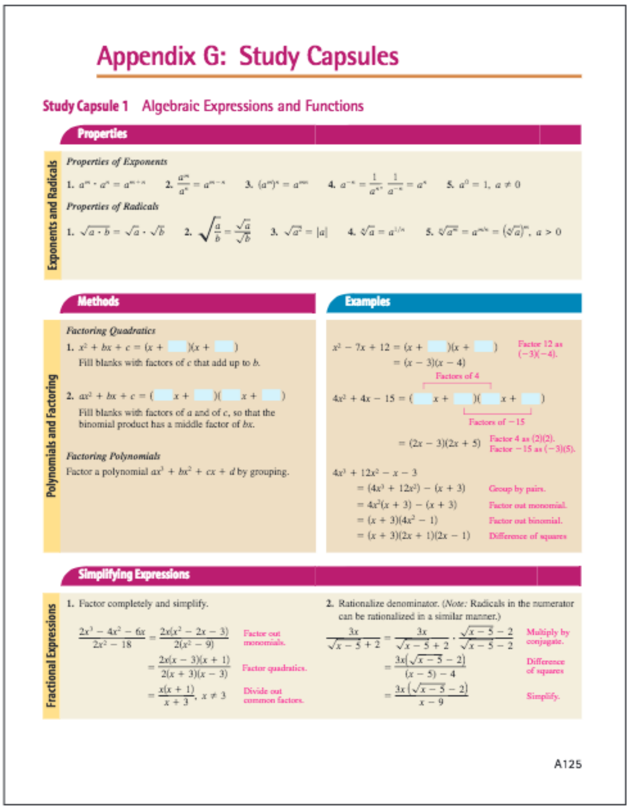
Each *Study Capsule* in Appendix G summarizes many of the key concepts covered in previous chapters. A *Study Capsule* provides definitions, examples, and procedures for solving, simplifying, and graphing functions. Students can use this appendix as a quick reference when working on homework or studying for a test.


256 Chapter 3 Exponential and Logarithmic Functions

Progressive Summary (Chapters 1–3)

This chart outlines the topics that have been covered so far in this text. Progressive Summary charts appear after Chapters 2, 3, 6, 9, and 11. In each progressive summary, new topics encountered for the first time appear in red.

Algebraic Functions	Transcendental Functions	Other Topics
Polynomial, Rational, Radical	Exponential, Logarithmic	
■ Rewriting Polynomial form ↔ Factored form Operations with polynomials Rationalize denominators Simplify rational expressions Operations with complex numbers	■ Rewriting Exponential form ↔ Logarithmic form Condense/expand logarithmic expressions	■ Rewriting
■ Solving *Equation* — *Strategy* Linear — Isolate variable Quadratic — Factor, set to zero; Extract square roots; Complete the square; Quadratic Formula Polynomial — Factor, set to zero; Rational Zero Test Rational — Multiply by LCD Radical — Isolate, raise to power Absolute value — Isolate, form two equations	■ Solving *Equation* — *Strategy* Exponential — Take logarithm of each side Logarithmic — Exponentiate each side	■ Solving
■ Analyzing *Graphically*: Intercepts, Symmetry, Slope, Asymptotes, End behavior, Minimum values, Maximum values *Algebraically*: Domain, Range, Transformations, Composition, Standard forms of equations, Leading Coefficient Test, Synthetic division, Descartes's Rule of Signs *Numerically*: Table of values	■ Analyzing *Graphically*: Intercepts, Asymptotes *Algebraically*: Domain, Range, Transformations, Composition, Inverse properties *Numerically*: Table of values	■ Analyzing


Appendix G: Study Capsules

Study Capsule 1 Algebraic Expressions and Functions

Properties

Exponents and Radicals

Properties of Exponents

1. $a^m \cdot a^n = a^{m+n}$ 2. $\frac{a^m}{a^n} = a^{m-n}$ 3. $(a^m)^n = a^{mn}$ 4. $a^{-n} = \frac{1}{a^n}, \frac{1}{a^{-n}} = a^n$ 5. $a^0 = 1, a \neq 0$

Properties of Radicals

1. $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$ 2. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ 3. $\sqrt{a^2} = |a|$ 4. $\sqrt[n]{a} = a^{1/n}$ 5. $\sqrt[n]{a^m} = a^{m/n} = (\sqrt[n]{a})^m, a > 0$

Polynomials and Factoring

Methods

Factoring Quadratics

1. $x^2 + bx + c = (x + \square)(x + \square)$
Fill blanks with factors of c that add up to b.

2. $ax^2 + bx + c = (\square x + \square)(\square x + \square)$
Fill blanks with factors of a and of c, so that the binomial product has a middle factor of bx.

Factoring Polynomials

Factor a polynomial $ax^3 + bx^2 + cx + d$ by grouping.

Examples

$x^2 - 7x + 12 = (x + \square)(x + \square)$ Factor 12 as $(-3)(-4)$.
$= (x - 3)(x - 4)$
Factors of 4

$4x^2 + 4x - 15 = (\square x + \square)(\square x + \square)$
Factors of −15

$= (2x - 3)(2x + 5)$ Factor 4 as (2)(2). Factor −15 as (−3)(5).

$4x^3 + 12x^2 - x - 3$
$= (4x^3 + 12x^2) - (x + 3)$ Group by pairs.
$= 4x^2(x + 3) - (x + 3)$ Factor out monomial.
$= (x + 3)(4x^2 - 1)$ Factor out binomial.
$= (x + 3)(2x + 1)(2x - 1)$ Difference of squares

Simplifying Expressions

Fractional Expressions

1. Factor completely and simplify.

$\frac{2x^3 - 4x^2 - 6x}{2x^2 - 18} = \frac{2x(x^2 - 2x - 3)}{2(x^2 - 9)}$ Factor out monomials.
$= \frac{2x(x - 3)(x + 1)}{2(x + 3)(x - 3)}$ Factor quadratics.
$= \frac{x(x + 1)}{x + 3}, x \neq 3$ Divide out common factors.

2. Rationalize denominator. (*Note:* Radicals in the numerator can be rationalized in a similar manner.)

$\frac{3x}{\sqrt{x - 5} + 2} = \frac{3x}{\sqrt{x - 5} + 2} \cdot \frac{\sqrt{x - 5} - 2}{\sqrt{x - 5} - 2}$ Multiply by conjugate.
$= \frac{3x(\sqrt{x - 5} - 2)}{(x - 5) - 4}$ Difference of squares
$= \frac{3x(\sqrt{x - 5} - 2)}{x - 9}$ Simplify.

A125

FEATURES

Additional Resources—Get the Most Out of Your Textbook!

Supplements for the Teacher

Teacher's Annotated Edition

Complete Solutions Guide

Online Complete Solutions Guide

Electronic Classroom

Online Instructor Success Organizer

Online Teaching Center

This free companion website contains an abundance of teacher resources. Visit **www.cengage.com/math/larson/precalclim_aga5e** and click on the Online Teaching Center icon.

Diploma Testing
"Testing the way you want it"

Diploma Testing provides instructors all the tools they need to **create, author/edit, customize,** and **deliver** multiple types of tests. Teachers can use existing test bank content, edit the content, and author new static or algorithmic questions—all within Diploma's powerful electronic platform.

Supplements for the Student

Study and Solutions Guide

Written by the author, this manual offers step-by-step solutions for all odd-numbered text exercises as well as Chapter and Cumulative Tests. The manual also provides practice tests that are accompanied by a solution key. In addition, these worked-out *Notetaking Guide* solutions are available at *www.CalcChat.com*.

Online Study Center

This free companion website contains an abundance of student resources including the *Online Student Notetaking Guide*. Visit the website **www.cengage.com/math/larson/precalclim_aga5e** and click on the Online Study Center icon.

Instructional DVDs

Hosted by Dana Mosely, these text-specific DVDs cover all sections of the text and provide key explanations of key concepts, examples, exercises, and applications in a lecture-based format. New to this edition, the DVDs will now include Captioning for the Hearing Impaired.

Eduspace®: Cengage Learning's Online Learning Tool (Powered by Blackboard™)

Eduspace is a web-based learning system that provides instructors and students with powerful course management tools and text-specific content to support all of their online teaching and learning needs. By pairing the widely recognized tools of Blackboard with customizable content from Cengage Learning, Eduspace makes it easy to deliver all or part of a course online. Teachers can use Eduspace to offer a combination of ready-to-use resources to students. This online learning tool contains ready-to-use resources such as algorithmic and non-algorithmic homework, quizzes, tests, tutorials, instructional videos, interactive textbooks, live online tutoring with **SMARTHINKING®**, and additional study materials. Teachers can choose to use the content as is, modify it, or even add their own content. Visit *www.eduspace.com* for more information.

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Deliver program or text-specific Cengage Learning content online using your institution's local course management system. Cengage Learning offers homework, tutorials, videos, and other resources formatted for Blackboard™, WebCT®, eCollege®, and other course management systems. Add to an existing online course or create a new one by selecting from a wide range of powerful learning and instructional materials.